only the first few terms of this series need be evaluated.

$$\frac{4a}{\pi^2 d^2}\int_0^r \sin\left(\frac{2\pi}{a}\sqrt{r^2-y^2}\right)dy$$
$$=\frac{4a}{\pi^2 d^2}\left[\frac{\pi^2 r^2}{2a}-\frac{\pi^4 r^4}{4a^3}+\frac{\pi^6 r^6}{23a^5}-\cdots\right]$$
$$=\frac{1}{2}-\frac{1}{4}\left(\frac{\pi d}{2a}\right)^2+\frac{1}{24}\left(\frac{\pi d}{2a}\right)^4, \qquad d/a<0.4. \qquad (A6)$$

Substituting (A6) in (A4), the end-wall field-averaging correction factor is obtained

$$c_2^e=1-\frac{1}{4}\left(\frac{\pi d}{2a}\right)^2+\frac{1}{24}\left(\frac{\pi d}{2a}\right)^4, \qquad d/a<0.4. \qquad (A7)$$

Side-wall iris

At the iris, the incident field $\underline{H}_t$ consists entirely of the $\underline{H}_z$ component, and since the aperture is located at $x=0$

$$H_z - jH_0\frac{\lambda g}{2a}\sin\frac{\pi z}{l_1}. \qquad (A8)$$

Substituting in (A1), it is readily shown that when $\epsilon = l_{1/2}$

$$c_2^s=1-\frac{1}{4}\left(\frac{\pi d}{2l_1}\right)^2+\frac{1}{24}\left(\frac{\pi d}{2l_1}\right)^4, \qquad d/l_1<0.4. \qquad (A9)$$

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A Technique to Identify Electromagnetic Modes in Oversize Waveguides

Y. CARMEL, K. R. CHU, M. E. READ, V. L. GRANATSTEIN
G. FAILLON, P. BOULANGER, E. KAMMERER,
AND G. MOURIER

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Y. Carmel and V. L. Granatstein are with the Laboratory for Plasma and Fusion Energy Studies, University of Maryland, College Park, MD 20742.

K. R. Chu and M. E. Read are with the Naval Research Laboratory, Code 4740, Washington, DC 20375.

G. Fallon, P. Boulanger, E. Kammerer, and G. Mourier are with Thomson–C.S.F., Division Tubes Electroniques, 92102 Boulange—Billancourt Cedex, France.

***Abstract*—There are problems associated with the multimode character of oversize waveguides. This paper reports on a novel, direct way to identify modes in an oversize waveguide by looking at the field map on a liquid crystal sheet inserted in the waveguide. The local temperature change in the liquid crystal due to absorbed microwave energy is translated to a color change in such a way that a map of the local power flow is observed on the sheet.**

Mode identification is very important in gyrotrons, for example, where the microwave energy generating device is subject to severe problems from mode competition.

I. Introduction

It is well known that different modes of electromagnetic waves in a waveguide have different attenuation for a waveguide of fixed size. For example, a $TE_{0,1}^0$ (transverse electric in circular guide) mode has decreasing attenuation with increasing frequency. This property is shared by all $TE_{0,n}$ modes, and, because of this property, some waves have received a good deal of attention for possible long-distance propagation of energy [1], and for low ohmic losses in resonators of microwave and millimeter-wave power sources such as gyrotrons [2]. One of the major problems in the use of $TE_{0,n}$ in circular waveguides arises because it is not the mode of lowest cutoff frequency, and therefore must always be used in a guide capable of propagating a number of modes (known as oversize guides). For the $TE_{0,1}$ mode in a circular waveguide, there are at least four other modes propagating if the $TE_{0,1}$ mode is above cutoff ($TE_{1,1}$, $TM_{0,1}$, $TE_{2,1}$). If the $TE_{0,4}$ mode is above cutoff, at least 48 modes can propagate in the guide.

The practical problems raised by the multimode character are several. In the first place, one must have a method of exciting and identifying the desired mode. This is very important in gyrotrons, for example, where the microwave-generating device is subject to severe problems from mode competition [3], [4]. Specifically, the device can oscillate in an undesired mode having a resonance frequency close to that of the desired mode. Secondly, one must guard against coupling from the desired mode, once excited, to undesired modes. Thirdly, the measurement of power propagating in an oversize waveguide by a high-order mode wave is not simple, in contrast to power measurement in the fundamental frequency which is a routine matter. There are few ways to identify the mode of propagation in an oversize waveguide, and even to measure the fractional power in each of them. A comparison of several of the measurement methods is presented in [5] and [6].

This paper report on another, direct way to identify modes by looking at the field map on a liquid crystal sheet. This technique was first suggested by G. Faillon, following a method described in the literature, especially with regard to the study of antenna patterns [7]–[10].

II. Description of the Technique

Plastic-encapsulated liquid crystal sheets are heat sensitive, and if microwave energy is absorbed by the sheet it results in local temperature change. This temperature change appears as a local color change in such a way that a map of the RF field E (more exactly of $|E|^2$ or local power flow) can be observed on the sheet. If the liquid crystal sheet is inserted in a waveguide or horn, the mode pattern will be clearly visible. This technique is especially useful in, but not limited to, oversize waveguides in which a large number of modes can propagate.

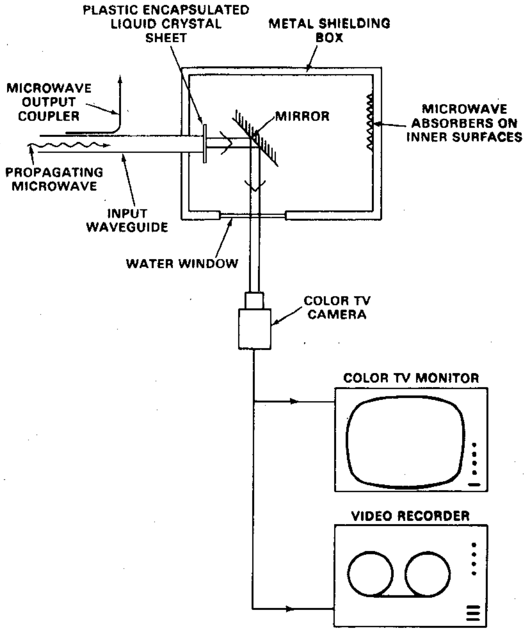

Fig. 1. A schematic diagram of the system used to identify modes using the liquid crystal technique.

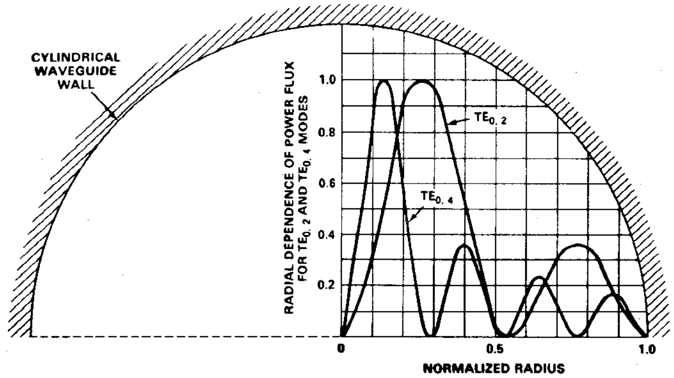

Fig. 2. The radial dependence of power flux for $TE_{0,2}$ and $TE_{0,4}$ modes propagating in a cylindrical waveguide.

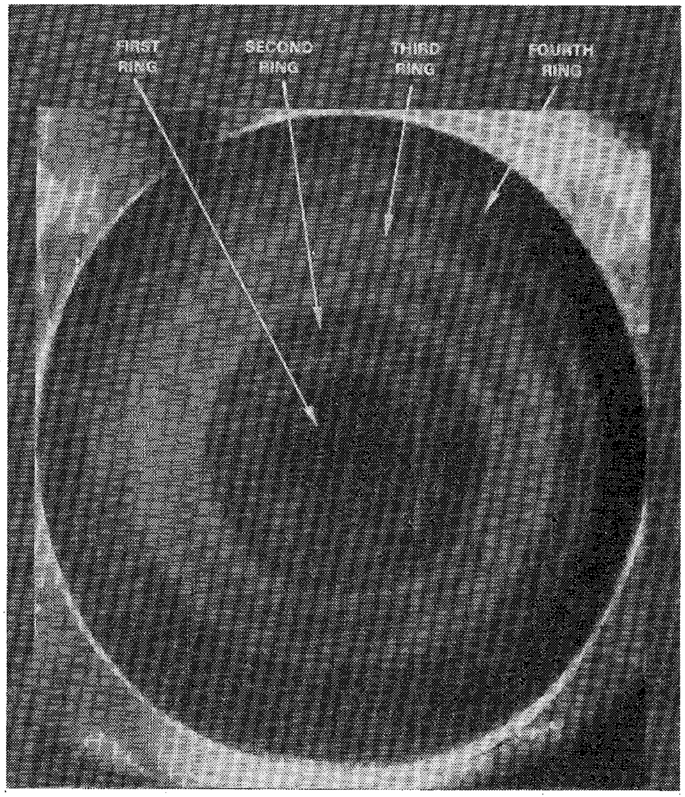

Fig. 3. A selected video frame showing the mode pattern of a $TE_{0,4}$ gyrotron as observed on the liquid crystal sheet. The colors are: blue (first ring), blue-green (separator ring), blue (second ring), brown (separator), yellow-orange (third ring), brown (separator), and rust (fourth ring).

A schematic diagram of the system used to identify modes at the output of a 35-GHz gyrotron tube is given in Fig. 1. When 8.5-mm radiation is propagating in a cylindrical oversize waveguide having a diameter of 36.53 mm, 74 modes can possibly propagate in such a guide. The liquid crystal sheet is mounted perpendicular to the guide axis at its output. Being almost transparent to the electromagnetic radiation, the sheet does not perturb the field profile in the guide. In the experiment at Thomson-CSF, the tube output window, 5 cm in diameter, was followed by a 26-cm-long cone terminating with a diameter of 12 cm. At the end of this cone, the wave propagates very nearly as a plane wave and can be launched into free space almost without reflection. For the protection of the personnel, the tube was installed inside a large bunker, and only the neighboring accessories were provided with absorbing material. A color-TV camera was placed off axis at about 1 m from the horn; thus, permitting to continuously monitor the mode pattern and, for instance, to observe mode transitions as a function of magnetic field.

In the NRL experiments, a color-TV camera is used to continuously monitor the mode pattern on the crystal through a mirror. In order to protect the personnel from electromagnetic radiation hazards,[1] the liquid crystal sheet and the mirror are enclosed in a metallic shielding box and the optical image is transmitted through a water window which is optically transparent but opaque to microwaves. The inner walls of the shielding box are lined with microwave-absorbing material in order to minimize internal electromagnetic reflections. Such reflections, if present, might alter the field's map at the cylindrical aperture where the image is observed. The image is continuously recorded by a video taperecorder (also monitored on a standard TV monitor). The continuous TV monitoring of the liquid crystal sheet is essential since the image is dynamic. This is because the electromagnetic power flow is perpendicular to the sheet, but heat transfer due to conduction is transverse (parallel to the surface). It is that transverse heat diffusion which eventually causes the image to be smeared or, in other words, causes the sheet to reach a steady-state, quasi-uniform temperature distribution. By continuously monitoring the image evolution, we can pick up a selected video frame taken before steady state was established.

The liquid crystal sheet has a dynamic range of about 5°C,[2] and the colors change from brown (cold) to violet (hot) through red, yellow, green, and blue, in that order.

III. Application of the Technique

The technique will be illustrated by identifying TE_{mn} modes in a cylindrical waveguide. Five field components are present, in general, for $TE_{m,n}$ modes ($E_r, E_\theta, B_z, B_r, B_\theta$). The radial dependence of power flow (in the z direction) is calculated by

$$P \alpha \hat{e}_z \cdot E \times B^* = E_r B_\theta^* - E_\theta B_r^*$$

[1]The U.S. Standard allows for exposure to 1 mW/cm^2.

[2]Parker Liquid Crystal Ink on Plastic, Mode 72 375 (20–20°C).

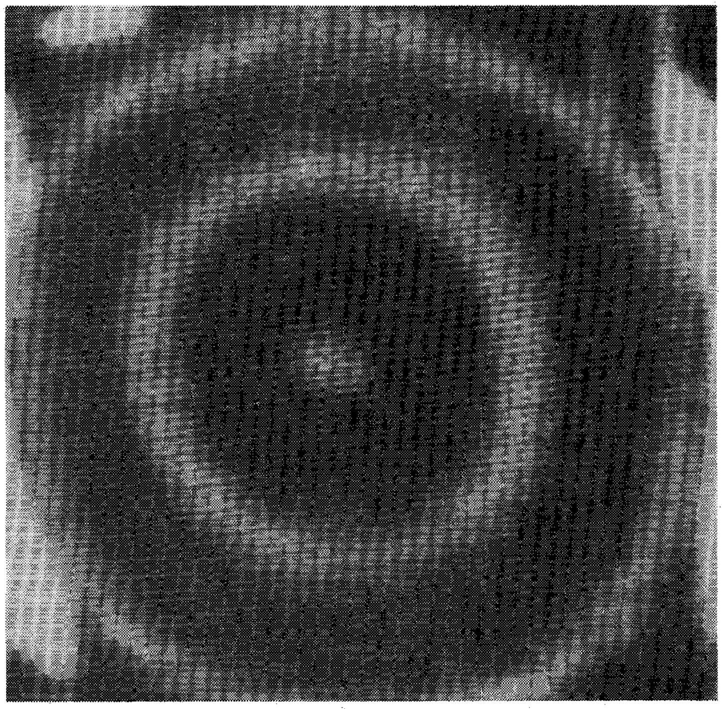

Fig. 4. Same as Fig. 3, but for a $TE_{0,2}$ mode. The two dark rings are blue to blue-green and indicate the peaks in the radiation. The light rings are yellow, and indicate nulls.[3]

where all the field components have time dependence of the form

$$e^{im\theta + ik_z z - i\omega t}$$

where k_z is the axial wavenumber, ω is the angular frequency, t is the time, and θ is the angle. In Fig. 2, we see the radial dependence of power flux for two modes ($TE_{0,2}$, $TE_{0,4}$).

$TE_{0,4}$ has four peaks between the center and the inner wall of the waveguide, while the $TE_{0,2}$ has two. It is therefore expected that the two modes will have four and two concentric rings on the field map, as is clearly visible in Fig. 3 and 4, respectively. Different modes can be identified using the same arrangement and technique—for example $TE_{2,4}$ and $TE_{12,1}$ have been identified. Also, asymmetry in the mode can be detected.

The Thomson–CSF gyrotron is designed to oscillate in the TE_{02} mode with a magnetic field of about 13.5 kG. Fig. 4 shows the pattern corresponding to this mode in the correct conditions. At a lower field, the tubes often oscillate in the $TE_{2,2}$ mode, theoretically with a rotating field pattern. The $TE_{0,n}$ modes are certainly the easiest to identify, as they are not degenerate except with TM modes that are very difficult to excite in gyrotrons.

The power flux needed to create a clear, high-quality image on the liquid crystal at (35 GHz) is on the order of 700 mW/cm^2. Of course, only a very small fraction of that power is absorbed in the liquid crystal sheet. (Based on the liquid crystal sheet material, it is estimated that only about 10^{-3} of the electromagnetic energy flux is absorbed at 35 GHz.)

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Waveguide Modes Via an Integral Equation Leading to a Linear Matrix Eigenvalue Problem

GIUSEPPE CONCIAURO, MARCO BRESSAN, AND CINZIA ZUFFADA

Abstract **—A numerical method for determining the modes of a rectangular or a circular waveguide strongly perturbed by axial cylindrical conduct-**

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The authors are with the Dipartimento di Elettronica dell'Universitá di Pavia, 27100 Pavia, Italy.

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